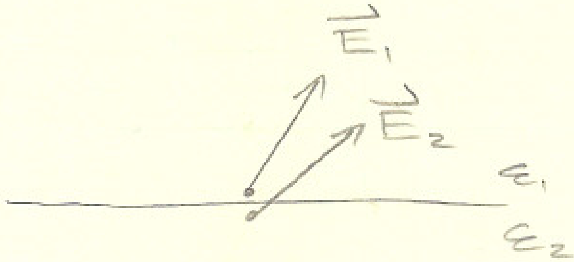
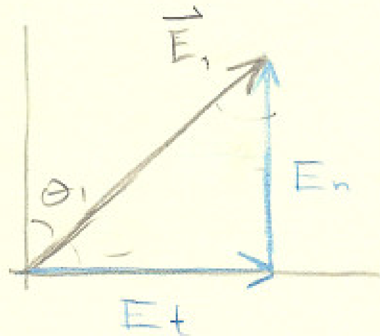
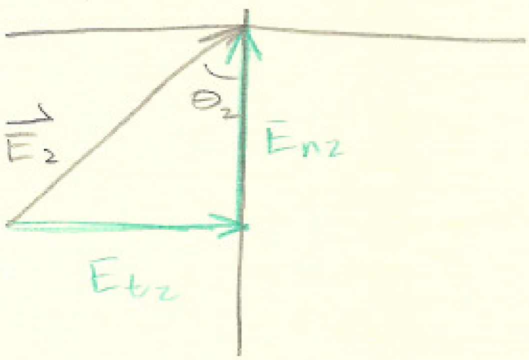
Dialectric to Dialectric interface



$$\vec{E}_{t1} = \vec{E}_1 \sin\theta_1$$

$$\vec{E}_{n1} = \vec{E}_1 \cos\theta_1$$





$$\vec{E}_{t2} = \vec{E}_2 \sin\theta_2$$

$$\vec{E}_{n2} = \vec{E}_2 \cos\theta_2$$

$$\vec{E}_{2t} = \vec{E}_{1t}$$

$$\vec{E}_2 \sin\theta_2 = \vec{E}_1 \sin\theta_1$$

$$D_{1n} - D_{2n} = \sigma_{free}$$

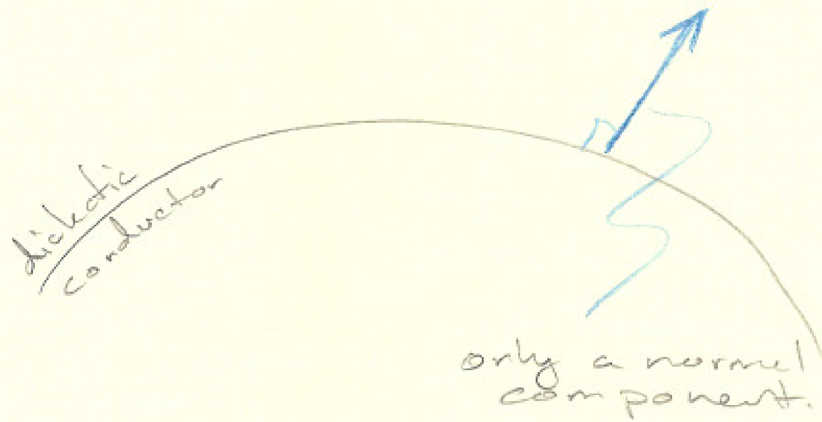
Assume free ($\sigma_{free} = \emptyset$) charge equal to zero.

$$\boxed{\tan\theta_2 = \frac{\varepsilon_2}{\varepsilon_1}\tan\theta_1}$$

$$\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$$

$$\varepsilon_1 \vec{E}_1 \cos\theta_1 = \varepsilon_2 \vec{E}_2 \cos\theta$$

$$\frac{1}{\varepsilon_1}\tan\theta_1 = \frac{1}{\varepsilon_2}\tan\theta_2$$



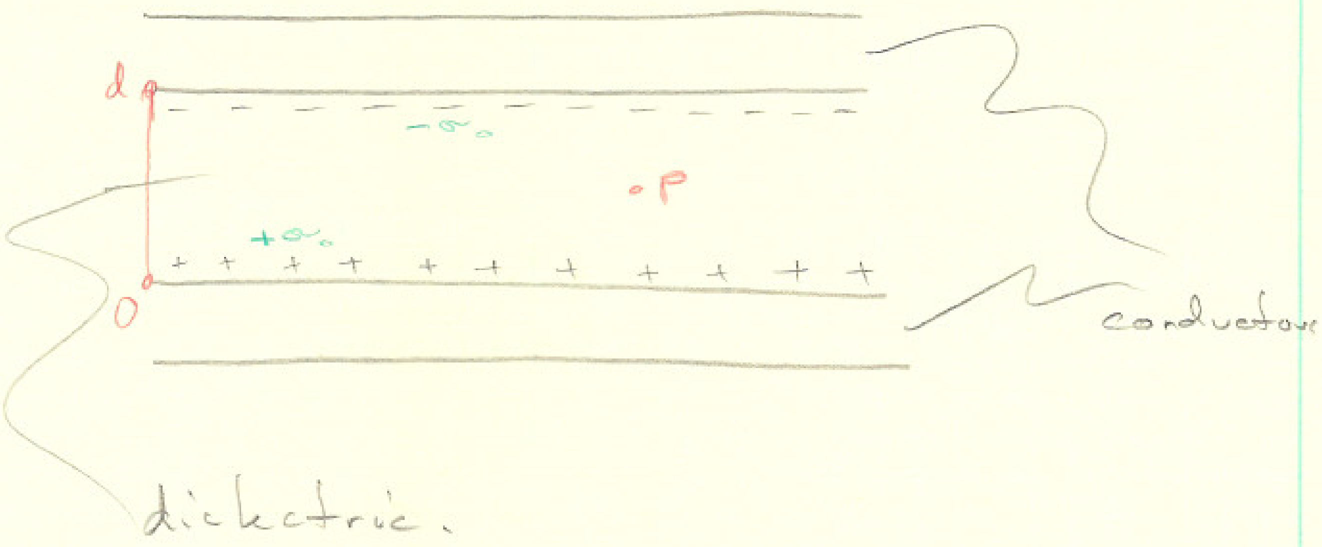
$$D_{n1} - D_{n2} = \sigma_{free}$$

$$\Downarrow$$

$$0$$

$$D_{n2} = \sigma_{free}$$

$$|E_{n2}| = \frac{\sigma_{free}}{\varepsilon}$$



$$|\vec{E}(P)| = \frac{\sigma_0}{2\varepsilon_0} - \left(-\frac{\sigma_0}{2\varepsilon_0}\right)$$

$$V_{0,d} = -\int_0^d \vec{E}\cdot d\vec{l} = \frac{-\sigma_0}{\varepsilon}\, d$$

$$C = \frac{Q}{V}$$

Charge on one of the bodies

Potential diff between plates

$$Q = \sigma_0 A$$

$$V = \frac{-\sigma_0 d}{\varepsilon}$$

$$C = \frac{A\varepsilon}{d}$$

Here we have just solved

$$Q \to \vec{E} \to V$$

But how do we go from

$$V \to \vec{E} \to Q$$

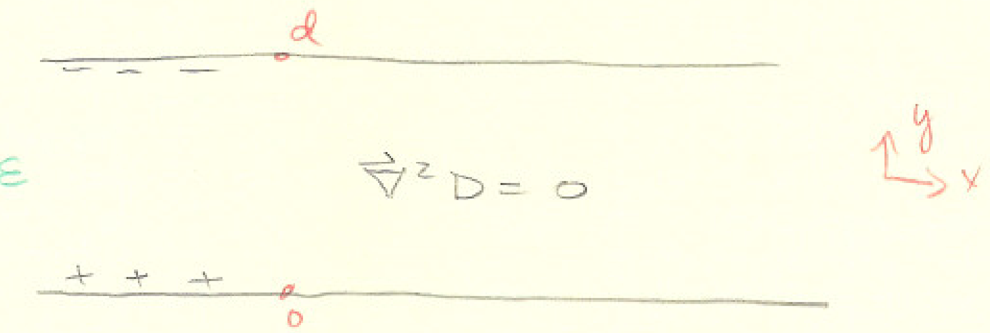
$$\vec{E} = -\vec{\nabla} \cdot \vec{D}$$

$$\oint \vec{D} \cdot d\vec{S} = \int \rho_{free}\, dv$$

$$\varepsilon \oint (\vec{\nabla} \cdot \vec{D})\, dv = \int \rho_{free}\, dv$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho_{free}}{\varepsilon} = \vec{\nabla}^2 \cdot V$$

$$\therefore \quad \vec{\nabla}^2 V = -\frac{\rho_{free}}{\varepsilon}$$



$$\vec{\nabla}^2 D = 0$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) V = 0$$

By Sym $V = V(y)$

$$\therefore \quad \frac{d^2V}{dy^2} = \emptyset$$

$$\therefore \quad \frac{dV}{dy} = k$$

constants

$$V(y) = ky + \alpha$$

$$V(0) = \emptyset \Rightarrow \alpha = 0$$

$$V(d) = kd = V_0$$

$$\therefore \quad k = \frac{V_0}{d}$$

$$\vec{E} = -\vec{\nabla} \cdot V$$

$$= \frac{-V_0}{d}$$

from $D_{1n} - D_{2n} = \sigma_{free}$

$$E_{2n} = -\varepsilon \sigma_{free}$$

$$\therefore \quad \frac{-V_0}{d} = -\frac{\sigma_{free}}{\varepsilon}$$

$$Q = \sigma_{free} A = \frac{V_0 \varepsilon}{d} \cdot A$$

$$C = \frac{Q}{V_0} = \frac{\varepsilon A}{d}$$

Which is the same as before.